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SPECULATIVE INVENTORY HOLDING AND PRICE STABILITY

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I. INTRODUCTION

As is true for many economic phenomena, the nature and effects of speculative carryover can be investigated from two polar viewpoints: perfect competition and monopoly. The competitive speculator does not expect his actions to affect current or future prices. He forms expectations about future prices on the basis of current and past prices and on possibly other information and then acts accordingly. The monopolistic speculator understands that his actions affect current and future prices--not only because his excess demand is part of aggregate excess demand but also because his actions may result in altered expectations on the part of other participants in the market.

The latter problem is undoubtedly the more interesting one. However it is important that we fully understand the competitive case as well. An understanding of the welfare properties of the competitive case is necessary for normative discussions concerning what types of behavior we would like to encourage when designing institutions. As well, a very interesting situation to analyze theoretically occurs when one or more monopolistic speculators operate simultaneously with competitive speculators. This "wolves preying on sheep" scenario is often thought to occur in the real world. Obviously understanding behavior of the sheep in the absence of the wolf is a useful first step in the larger problem; it also becomes a base case with which to compare further results and from which to gain further intuition.

Very little theoretical research on any aspect of the problem exists. A series of early papers [1, 7, 10, 11] consider the case where non-speculative excess demand is an affine function of current price. They prove that if speculators operating over some finite period of time earn positive *ex post* profits that prices were *ex post* stabilized over that period of time. This does not, however, formally justify Milton Friedman's famous remark.

> People who argue that speculation is generally destabilizing seldom realize that this is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators on the average sell when the currency is low in price and buy when it is high.[1]

A single monopolist speculator with perfect foresight would only engage in profitable and thus stabilizing speculation. However, speculators who misjudge their competitors' actions or who are

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not able to perfectly predict some exogenous randomness might earn negative ex post profits. These speculators could remain in the market over the long term so long as they earned positive average profits. Therefore if we want to conclude that speculators we are likely to find in a market are stabilizing, we need to prove that speculation which yields positive expected profits is stabilizing.

Hart [5] has recently considered the more general case where current excess demand is an affine function of current and past prices. This constitutes the first theoretical investigation of the "wolf and sheep" case; I will demonstrate that this excess demand function could be caused by the behavior of competitive speculators using past prices to predict future prices. He does not consider welfare properties or price stability; rather, he characterizes the cases where the wolf could make positive profits.

Only three papers to my knowledge have explicitly considered the case of competitive speculative carryover. Of these, only Kohn [8] explicitly calculates and uses optimal behavior for the speculators. He is also one of the few researchers to depart from the framework of non-speculative excess demand being affine. However, he pays a high price; he can only prove a limited existence and uniqueness theorem. Futia [3] solely considers the extent to which prices convey information in a market with competitive speculators where non-speculative excess demand is affine in current price. While this is an important problem it is outside the scope of

this paper. Finally, Muth [9] considers speculative carryover in his paper on rational expectations. He provides a non-constructive proof that a long run rational expectations equilibrium exists and that prices are stabilized at it. However, he uses an arbitrary rule for speculators to follow and does not explicitly consider their expectations--it is producers operating with a one period supply lag whose expectations are considered.

The approach of this paper is most closely related to that of Muth; it represents a full treatment of Muth's case. Speculators' expectations and optimal behavior on their part are treated explicitly. Equilibrium price paths as well as their limit are constructively proven to exist for all expectations formation functions (EFF's) affine in current price. The relationship between four economic concepts is fully explored for the entire class of EFF's. The four concepts are:

(i) Stability: To what extent does the speculator stabilize or destabilize prices?

(ii) Profitability: How large are the speculator's average profits?

(iii) Accuracy: How accurate is the speculator's estimate of next period's price?

(iv) Responsiveness: How responsive is the speculator's estimate of next period's price to changes in this period's price?

Friedman's assertion is true in a very strong and appealing form in this model. A speculator making optimal decisions based on a fixed EFF has one of two effects. Either:

(1) Speculation does not change average price, reduces variance of prices, and results in a positive correlation between prices, the correlation becoming smaller as prices move further apart. That is, speculation smooths price variation around the mean by partially shifting peaks to adjacent periods. The speculator may earn negative, zero, or positive average profits.

(2) The speculator exhibits hoarding behavior, resulting in hyperinflation and eventual market collapse. Expected profits are infinitely negative in the limit.

That is, a speculator making optimal decisions never can generate a finite increase in the variance of prices over the long run; he either reduces variance or causes a hyperinflation with infinite variance and expected price in the limit.

This paper also proves that if the speculator has rational expectations (that is, if his expectation of next period's price is the actual expected value of next period's price) that two possible equilibria exist--one of each of the types outlined above. The "best" and "worst" consequences of speculative behavior emerge as two possible equilibria of the same rational expectations model. The stable rational expectations equilibrium is the one identified by Muth.

Stability and profitability are therefore somewhat related. However, a speculator earning more profits is not necessarily more stabilizing. In this sense of the word

"relation"--existence of a monotone relation--profitability, stability and correctness are mutually unrelated. Stability and responsiveness are monotonely related, however. A more responsive speculator always stabilizes prices less. If a speculator's expectations are responsive enough, he will cause a hyperinflation and destabilize prices.

II. THE SPECULATOR

Assume that the speculator incurs costs in holding inventories. To avoid dealing with the boundary condition that inventories must be non-negative, also assume that storing inventories yields benefits in addition to those arising from speculative gain.[2] Most papers in the literature simply ignore this problem [1, 3, 9, 10, 11]. Identifying conditions sufficient to ignore the problem is no worse a solution. At least in some instances the existence of benefits from holding inventories are common and unsurprising. Pioneer scholars in the field have recognized this point.

> Stocks of all goods possess a yield . . . and this yield which is a compensation to the holder of stocks, must be deducted from carrying costs proper in calculating net carrying cost. The latter can, therefore be negative or positive.[3]

> Storage of goods without direct renumeration and without expectation of price appreciation is to be observed in every retail store. A merchant might adopt the practice of buying today only what he could be sure of selling before

tomorrow, or before the next delivery day, but if he did so he would be unlikely to remain long in business; he must carry stocks beyond known immediate needs and take his return in general customer satisfaction.[4]

In terms of our simple model, we might imagine that our speculator (who is now also a merchant) is usually faced with a lag in deliveries from the producer near the start of each period, and so must have extra stocks on hand if he wishes to avoid delays in satisfying demand.

Net costs of inventory holding are then simply costs minus benefits. Assume that net costs can be approximated by a quadratic function. Let s_t represent stocks of inventories held over at time t for time t + 1.[5] Then for some constants b, c, and d we can write out net costs of holding s_t,

$$\frac{c}{2}(s_t - b)^2 + d \qquad (1)$$

Assume that the speculator pays these costs after the storage has been performed (i.e. he pays for holding s_t during period t + 1).

Consider the speculator's problem at period t. He knows his current inventory, s_{t-1}, the current price p_t, and his expectation of price during periods after t, E_{t+i} for $i = 1,2,3,\ldots$. He needs to choose a new inventory level to carry over for next period, s_t. To do this he implicitly also selects a plan for future inventory holdings. Let

$$\{s_{t+i}\}_{i=1}^{\infty}$$

be this plan where s_{t+i} can depend upon p_{t+i} and s_{t+i-1}.

Definition: Let $\mathscr{s} = \{s_{t+i}\}_{i=0}^{\infty}$ be a set of measurable functions whose range is the non-negative real line and whose domain is R_+^2.

Then $\mathscr{s}$ is a <u>period t strategy</u>.

Strategies will be compared by using the overtaking criterion on the expected discounted value of profits. Assume that the discount factor, β, is between 0 and 1. Let $\mathscr{s}$ be a period t strategy. Then the discounted expected profits from time t to time r asrising from $\mathscr{s}$ are

$$D(\mathscr{s},r) = E\left\{\sum_{i=t}^{r} \beta^{i-t}[p_i(s_{i-1} - s_i) - \frac{c}{2}(s_{i-1} - b)^2 - d]\right\}. \qquad (2)$$

<u>Definition</u>: $\mathscr{s}$ is an <u>optimal period t strategy</u> if for any period t strategy $\mathscr{r}$, there exists an integer $r(\mathscr{r}) \geq t$ such that for every integer j greater than r

$$D(\mathscr{s},j) \geq D(\mathscr{r},j).$$

Because the speculator is competitive he believes his actions do not affect prices--the optimum strategy is simply the period by period optimum. That is, the speculator chooses s_t as if he were going out of business at the end of period t + 1. Kohn [8] discusses the intuition of this result. Note that the following proof is more general than his in that it is based on the overtaking criterion and can thus handle unbounded returns. As well, to use Kohn's dynamic programming approach we must assume that the state space of current price and inventories is compact; this is not a natural assumption. A priori, inventories or price might be arbitrarily large.

Proposition 1: The unique optimal period t strategy is

$$s_{t+i} = \max[0, \frac{E_{t+i+1}}{c} + b - \frac{p_{t+i}}{\beta c}]. \qquad (3)$$

Proof: Rewrite $D(\delta,\infty)$ as

$$D(\delta,\infty) = p_t s_{t-1} - \frac{c}{2}(s_{t-1} - b)^2 - d$$

$$+ \sum_{i=t}^{\infty} \delta i(s_i)$$

where

$$\delta_i(s_i) = \beta^{i-t}[s_i(\beta p_{i+1} - p_i) - \frac{\beta c}{2}(s_i - b)^2 - d].$$

Clearly if we could choose s_i to uniquely maximize δ_i, it would be the unique optimal strategy. The only question is whether an s_i chosen in this fashion is only a function of p_i and s_{i-1}. Expression (3) is the unique maximum to δ_i and it is only a function of p_i.

□

III. THE MODEL WITHOUT SPECULATORS

Let excess demand at time t be given by

$$-ap_t + \gamma_t \qquad (4)$$

where a is a positive constant and γ_t is independently and identically distributed across time with expected value Γ and variance V. Then in the absence of speculation, equilibrium price in time t is equal to γ_t/a. Therefore the expected value of p_t is Γ/a and the variance of p_t is V/a^2. Furthermore prices are independent across periods.

IV. LINEAR EXPECTATIONS

Assume that there is one speculator in the market. The results are immediately generalizable to case of n identical speculators. Assume that the speculator's expectations about next period's price are a linear function of last period's price,

$$E_t = \delta p_{t-1} + \varepsilon. \qquad (5)$$

Also assume that $\delta > 0$; a larger price generates expectations of a larger price. Equation (5) can be rewritten to yield a particularly natural interpretation,

$$E_t = \frac{\varepsilon}{1-\delta} + \delta\left(p_{t-1} - \frac{\varepsilon}{1-\delta}\right). \tag{6}$$

The speculator generally expects price to be around $\varepsilon/1 - \delta$. If p_{t-1} is $\varepsilon/1 - \delta$, so is E_t. If price in period t - 1 varies from $\varepsilon/1 - \delta$, the speculator adds or subtracts δ times the difference to his estimate. If δ is greater than 1, the speculator adds more than the difference; his expectation of p_t is always further from $\varepsilon/1 - \delta$ than is p_{t-1}. The speculator is "panicked" in the sense that deviations from $\varepsilon/1 - \delta$ in p_{t-1} cause even larger deviations in his expectations about p_t. Such expectations will be called responsive. In general, we will call expectations using a higher value for δ more responsive.

Substitute (6) into (3) to yield

$$s_t = \max[0, (\frac{\delta}{c} - \frac{1}{\beta c})p_t + \frac{\varepsilon}{c} + b]. \tag{7}$$

Equilibrium price at period t is determined by

$$-ap + \gamma_t + s_t = s_{t-1}. \tag{8}$$

Substitute from (7) into (8), assuming for the moment that $(\frac{\delta}{c} - \frac{1}{\beta c})p_t + \frac{\varepsilon}{c} + b$ is non-negative. This yields

$$p_t = \frac{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right)}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a} p_{t-1} - \frac{\gamma_t}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a}. \tag{9}$$

Rewrite this as

$$p_t = \mu p_{t-1} + \nu_t, \tag{10}$$

where

$$\xi = \frac{\delta}{c} - \frac{1}{\beta c} \tag{11}$$

$$\mu = \frac{\xi}{\xi - a} \tag{12}$$

$$\nu_t = \frac{-\gamma_t}{\xi - a} \tag{13}$$

This is a first order difference equation. Its solution is

$$p_t = \sum_{i=1}^{t} \mu^{t-i}\nu_i + \mu^t p_0 \tag{14}$$

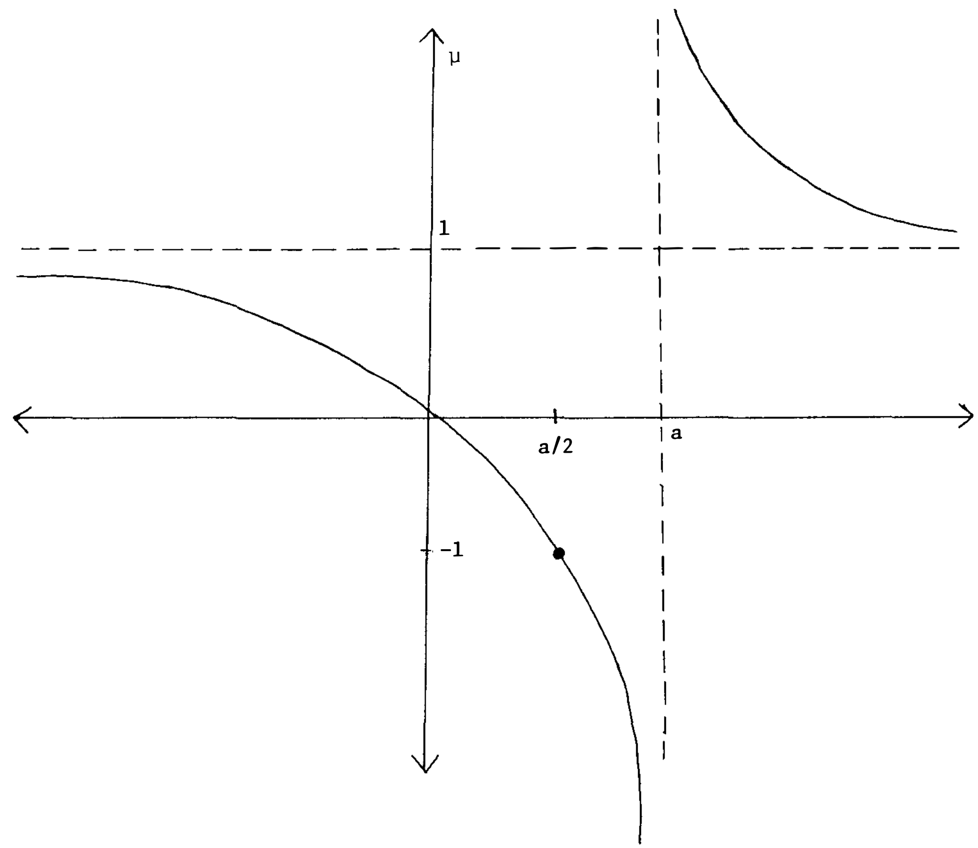
where p_0 is the initial value.[6] That is, if at period 0 the speculator were using the EFF (5) and we observed p_0, then (13) would give the price path from that period on if he continued to use (5).

A sufficient condition for $(\frac{\delta}{c} - \frac{1}{\beta c})p_t + \frac{\varepsilon}{c} + b$ to be non-negative, as assumed above, is that ξ is less than $\frac{a}{2}$ or bigger than a. In particular, the analysis applies to the case of unresponsive expectations formation mechanisms as well as moderately responsive ones. These, perhaps, are economically the most plausible cases. We also can analyze wildly responsive expectations formation mechanisms--those where ξ is greater than a. For the remainder of the paper we will only be considering expectations formation mechanisms such that ξ is less than $\frac{a}{2}$ or greater than a.

To see this, consider the graph of μ as a function of ξ in Figure 1. First suppose that ξ is less than $\frac{a}{2}$. Then μ is between 0 and 1 in absolute value. Assume that γ is always within $[0,\gamma^*]$ for some positive real γ^*. Then ν_t is bounded; by (10) p_t is also bounded. Consequently, b can be chosen large enough so that $(\frac{\delta}{c} - \frac{1}{\beta c})p_t + \frac{\varepsilon}{c} + b$ is always non-negative. We will always assume that where $\xi < a/2$, b is this large. That is, we will always assume that the benefits of holding inventories are large enough so that some small amount of inventories will always be held. Second consider the case where ξ is greater than a. Then μ is positive so that p_t is always positive and consequently $(\frac{\delta}{c} - \frac{1}{\beta c})\ p_t + \frac{\varepsilon}{c} + b$ is always positive. Therefore the only case where our assumption that $(\frac{\delta}{c} - \frac{1}{\beta c})p_t + \frac{\varepsilon}{c} + b$ is non-negative is inconsistent is when $\frac{a}{2} \leq \xi \leq a$. Here we have to consider the possibility of zero inventory carryover.

If we are interested in economies in which the speculator has been using the same EFF for a long time, the starting point, p_0, is arbitrary. The limit values of price are independent of p_0.

FIGURE 1



Proposition 2:

(i) $$\lim_{t\to\infty} E(p_t) = \begin{cases} \frac{\Gamma}{a}, & |\mu| < 1 \\ +\infty, & \mu > 1 \end{cases}$$

(ii) $$\lim_{t\to\infty} \mathrm{Var}(p_t) = \begin{cases} \frac{V}{a^2}\left(\frac{a}{a - 2\xi}\right), & |\mu| < 1 \\ \infty & |\mu| > 1 \end{cases}$$

(iii) $$\mathrm{Cov}(p_t, p_{t+n}) = \mu^n V(p_t)$$

Proof: I will present the proofs for $|\mu| < 1$. The other case should be clear.

(i) $$\lim_{t\to\infty} E(p_t) = \lim_{t\to\infty} E\left(\sum_{i=1}^{t} \mu^{t-i}\nu_i + \mu^t p_0\right)$$

$$= \frac{-\Gamma}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a} \sum_{i=0}^{\infty} \mu^i$$

$$= \frac{-\Gamma}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a} \cdot \frac{1}{1 - \mu}$$

$$= \frac{-\Gamma}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a} \cdot \frac{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a}{-a}$$

$$= \frac{\Gamma}{a}$$

(ii) $$\lim_{t\to\infty} \mathrm{Var}(p_t) = \lim_{t\to\infty} \mathrm{Var}\left[\sum_{i=1}^{t} \mu^{t-i}\nu_i + \mu^t p_0\right]$$

$$= \frac{V}{\left(\frac{\delta}{c} - \frac{1}{\beta c} - a\right)^2} \cdot \sum_{i=0}^{\infty} \mu^{2i}$$

$$= \frac{V}{\left(\frac{\delta}{c} - \frac{1}{\beta c} - a\right)^2} \cdot \frac{1}{1 - \mu^2}$$

$$= \frac{V}{\left(\frac{\delta}{c} - \frac{1}{\beta c} - a\right)^2} \cdot \frac{\left(\frac{\delta}{c} - \frac{1}{\beta c} - a\right)^2}{\left(\frac{\delta}{c} - \frac{1}{\beta c} - a\right)^2 - \left(\frac{\delta}{c} - \frac{1}{\beta c}\right)^2}$$

$$= \frac{V}{-2a\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) + a^2}$$

$$= \frac{V}{a^2} \cdot \frac{a}{a - 2\left(\frac{\delta}{c} - \frac{1}{\beta c}\right)}$$

(iii) $$\mathrm{Cov}(p_t, p_{t+n}) =$$

$$= \mathrm{Cov}\left[\sum_{i=1}^{t} \mu^{t-i}\nu_i + \mu^t p_0, \sum_{i=1}^{t+n} \mu^{(t+n-i)}\nu_i + \mu^{t+n} p_0\right]$$

$$= \sum_{i=1}^{t} \left(\mu^{2(t-i)+n} \mathrm{Var}(\nu_i)\right)$$

$$= \mu^n \mathrm{Var}\left(\sum_{i=1}^{t} \mu^{(t-i)}\nu_i\right)$$

$$= \mu^n \mathrm{Var}(p_t).$$

$\square$

Therefore when the speculator exhibits an unresponsive expectations formation mechanism, the variance of prices is reduced about the same mean by smoothing between periods. This results in a positive correlation between prices. All these effects become less pronounced, the more responsive is the expectations formations mechanism. These effects even continue when the expectations formations process is somewhat responsive. However, if the process becomes responsive enough, expected price goes to infinity. Essentially, observation of a high price triggers inventory holding which raises prices which generates further hoarding etc. The process continues until the market collaspses or, more likely, until speculators change their method of expectations formation.

Four remarks are in order. First, $\mathrm{Var}(p_t)$ converges upwards to its limit. Therefore introduction of a speculator results in a reduced variance in the short as well as the long run when $|\mu| < 1$.

Second, part (iii) of Proposition 2 constitutes a testable hypothesis which might be used to test the theory's applicability to individual markets. That is, we can check if a number μ exists such that the theory is applicable to that market. Note that the entire theory could be translated into terms of log linearity if this was thought to be more appropriate for econometric purposes.

Third, the numerical value of ε does not affect the conclusions of Theorem 1. A higher ε only means that inventories are perpetually higher by ε/c. This feature is a result of the quadratic cost function.

Fourth, the simplest case of linear expectations is constant expectations--where δ is 0. This is of course an unresponsive expectations function so all the comments concerning unresponsive expectations formation functions apply to the case where speculators always expect price to be a given constant.

V. RATIONAL EXPECTATIONS

One might expect that if speculators really used a constant expectation that over the long run ε would become Γ/a which is the unconditional expected value of price. In the class of constant expectations $\varepsilon = \Gamma/a$ is certainly the best choice.[7] However, the speculator is not limited to forming the same expectation period after period. In particular, the speculator can use current and past information to help predict next period's price. The speculator should only be satisfied with his method of prediction if his guess is the best possible using all past information. That is, it should be true that

$$E_t = E(p_t/\{p_i\}_{i=0}^{t-1},\{\gamma_i\}_{i=1}^{t-1}). \tag{15}$$

An EFF which produces this result is said to be rational, and the market is then said to be in a rational expectations equilibrium. This is also Muth's [9] and Kohn's [8] rational definition of rational expectations.

In the class of expectations of the form

$$E_t = \delta p_{t-1} + \varepsilon \tag{16}$$

there turn out to be precisely two choices for (δ,ξ) which yield rational expectations equilibria. To see this, substitute (5) and (9) into (15) to yield

$$\delta p_{t-1} + \varepsilon = \frac{\frac{\delta}{c} - \frac{1}{\beta c}}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a}\, p_{t-1} - \frac{\Gamma}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a} \tag{17}$$

Since this is an indentity (i.e. it is true for every value of p_{t-1}) we have

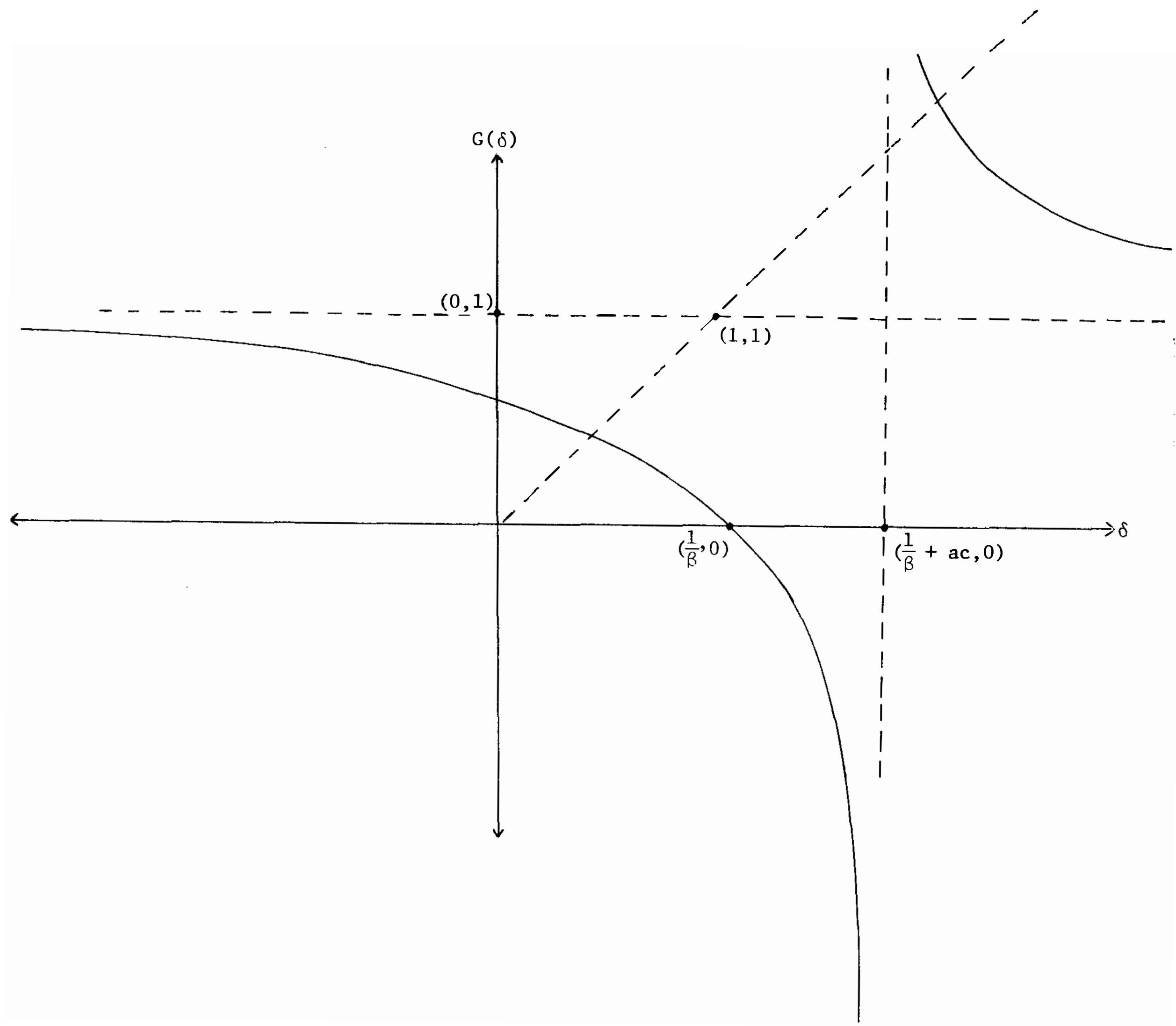
$$\delta = \frac{\frac{\delta}{c} - \frac{1}{\beta c}}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a} \tag{18}$$

$$\varepsilon = \frac{-\Gamma}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a} \tag{19}$$

Diagrammatically, view the RHS of (18) as a function of δ,

$$G(\delta) = \frac{\frac{\delta}{c} - \frac{1}{\beta c}}{\left(\frac{\delta}{c} - \frac{1}{\beta c}\right) - a}\ . \tag{20}$$

Figure 2



The function G is a hyperbola (see Figure 2). The two solutions to $G(\delta) = \delta$ are the two points of intersection of the hyperbolas with the 45° line. Each intersects once. Note that the intersection with the upper hyperbola occurs at a value of δ greater than $\frac{1}{\beta} + ac$. It can also be shown that the intersection with the lower hyperbola occurs at a value less than $\frac{1}{a\beta c + 1}$ (which is in particular less than one). We will use the notation δ_L and ε_L and δ_U and ε_U for the values associated with the lower and upper intersections.

Proposition 3:

$$\delta_L < \frac{1}{a\beta c + 1} .$$

Proof: It is clearly sufficient to prove that $G(\frac{1}{a\beta c + 1}) < \frac{1}{a\beta c + 1}$. We will now do this.

$$G\left(\frac{1}{a\beta c + 1}\right) = \frac{\frac{-1}{a\beta c^2 + c} + \frac{1}{\beta c}}{a + \left(\frac{1}{\beta c} - \frac{1}{a\beta c^2 + c}\right)}$$

$$= \frac{-\beta c + a\beta c^2 + c}{a(a\beta c^2 + c)\beta c - \beta c + a\beta c^2 + c}$$

$$= \frac{1}{1 + \frac{a\beta c^2(a\beta c + 1)}{a\beta c^2 + c(1 - \beta)}}$$

$$= \frac{1}{1 + a\beta c[\frac{a\beta c + 1}{a\beta c + 1 - \beta}]}$$

$$< \frac{1}{1 + a\beta c} .$$

$\square$

Therefore the expectations formation function using (δ_L,ε_L) is unresponsive and possesses all the properties of an unresponsive EFF discussed in section IV. The second rational EFF characterized by (δ_U,ε_U) is sufficiently responsive so that expected price and variance converge to infinity. That is, it is an example of self-fulfilling hoarding psychology producing a hyperinflation.

VI. UPDATING THE EFF

Aside from two problems "traditional" to the literature--linearity and negative inventories--the major weakness of this paper is probably the assumption that the speculator uses a fixed EFF. We might actually expect him to be constantly revising his EFF in the light of new information. The entire time path is calculated in the previous sections but the results following this section will only apply to a long run equilibrium situation.

The notion of rational expectations is of course meant to finesse this problem; there is no need for the speculator to revise his EFF when he is already using the "best" method available. However, the unstable rational expectations equilibrium seems unlikely to be long-lasting under a "reasonable" updating scheme. Presumably speculators would originally be quite happy with their use of the numbers $(\delta_U, \varepsilon_U)$ because their predictions would be on average correct. However as they began to notice that the large price rises were associated with equally large inventory buildups, they might use this information to revise their expectations downward and the "bubble" would burst. That is, the existence of a self-fulfilling hoarding psychology might tend to produce price bubbles in markets. Bubbles might occur when speculators were in the unstable rational expectations equilibrium and last until information about inventory buildups caused a downward revision in expectations. Conjectures such as this point out the need for an explicit treatment of the updating process.

VII. ACCURACY

We can use the limit as t goes to infinity of the variance of E_t around the conditional expected value of p_t as a measure of how accurate the speculator's expectations are. A lower value indicates a more accurate method of forming expectations. A rational expectations formation function of course has a value of 0 by definition. Accuracy is independent of stabilization even within the class of stable EFF's. That is, a more accurate method of forming expectations is not necessarily

always more or less stabilizing than a less accurate method. EFF's become more stabilizing as δ becomes smaller independent of ε; they become more accurate (at least locally) as δ approaches δ_L (letting ε be $-\Gamma/\xi - a$). Therefore we cannot use the accuracy of a speculator's expectations to infer anything about his stabilizing influence or vice versa.

VIII. PROFITABILITY

The speculator's profits for period t are calculated as follows.

$$\pi_t = p_t s_{t-1} - p_t s_t - \frac{c}{2}(s_{t-1} - b)^2 - d \quad (21)$$

That is, at time t he sells s_{t-1}, earning $p_t s_{t-1}$. He buys s_t at a cost of $p_t s_t$ and also pays his bill of $\frac{c}{2}(s_{t-1} - b)^2 + d$ for the holding of s_{t-1}. If the speculator forms expectations about next period's price via the rule (5) we saw that price converges towards the random variable with expected value and variance given in Proposition 2. We can calculate the value towards which expected profits converge by using this information.

The case where $|\mu| > 1$ should be clearest. Here expected price, variance and expected inventory carryover all converge to infinity. Since the speculator increases his inventories on average every period, expected profits converge to minus infinity. The formal argument will not be presented for this case. It is easy to derive. This leaves the case of $|\mu| < 1$.

Proposition 4: Suppose the speculator forms expectations via equation (4). Let $|\mu| < 1$. Then

$$\lim_{t\to\infty} E(\pi_t) = \left[\frac{\xi}{2a(a - 2\xi)(\xi - a)}\right] \left[2a - c\xi(\xi - a)\right] V$$

$$- \frac{c\xi^2}{2a^2}\Gamma^2 - \frac{c\xi\varepsilon}{a\beta}\Gamma - \frac{\varepsilon^2 c}{2\beta^2} - d \qquad (22)$$

Proof: We will calculate expected profits in two pieces. Part I will calculate expected profits excluding inventory holding costs. Part II will calculate expected inventory holding costs.

Part I: First calculate the expected value.

$$E[p_t S_{t-1} - p_t S_t]$$

$$= E[p_t(S_{t-1} - S_t)]$$

$$= E[p_t(\frac{E_t - E_{t+1}}{c} + \frac{p_t - p_{t-1}}{\beta c})] \text{ by (3)}$$

$$= E[p_t(p_{t-1} - p_t)\xi] \text{ by (5)}$$

$$= \xi[\mathrm{Cov}(p_{t-1}, p_t) - \mathrm{Var}(p_{t-1})].$$

Now we use Proposition 2 to calculate the limit of the expected value.

$$\lim_{t\to\infty} E[p_t S_{t-1} - p_t S_t]$$

$$= \frac{\xi(\mu - 1)}{a(a - 2\xi)} V$$

$$= \frac{\xi}{(\xi - a)(a - 2\xi)} V.$$

Note that the above is always non-negative. That is, before inventory holding costs, the speculator always makes a profit regardless of his expectations.

Part II: First calculate the expected value.

$$E[\frac{c}{2}(s_t - b)^2]$$

$$= E[\frac{c}{2}(\frac{E_{t+1}}{\beta} - \frac{p_t}{\beta c})^2] \text{ by (3)}$$

$$= E[\frac{c}{2}(\xi p_t + \frac{\varepsilon}{\beta})^2] \text{ by (5)}$$

$$= \frac{c}{2}\xi^2(\mathrm{Var}(p_t) + E(p_t)^2) + \frac{c\xi\varepsilon}{\beta} E(p_t) + \frac{\varepsilon^2 c}{2\beta^2} + d$$

Now we use Theorem 1 to calculate the limit of the expected value.

$$\lim_{t\to\infty} E[\frac{c}{2}(S_t - b)^2]$$

$$= \frac{c\xi^2}{2a(a-2\xi)} V + \frac{c\xi^2}{2a^2}\Gamma^2 + \frac{c\xi\varepsilon}{\beta a}\Gamma + \frac{\varepsilon^2 c}{2\beta^2} + d$$

The total can now be had by subtracting Part II from Part I.

□

The above expression gives expected profits as a function of the pair (δ,ε) used by the speculator in forming expectations. It turns out that to answer the questions we are interested in, it is sufficient to consider the function giving maximum expected profits for the number δ (the maximum being taken over all possible values for ε). This has a simpler functional form.

Corollary 1:

$$\max_{\varepsilon} \lim_{t\to\infty} E(\pi_t) = \frac{\xi}{2a(a-2\xi)(\xi-a)}[2a - c\xi(\xi - a)]V - d \qquad (23)$$

Proof: Simply calculate the first order conditions. This yields

$$\varepsilon = -\frac{\xi\beta}{a}\Gamma. \qquad (24)$$

Substitute this into the expected profits function.

□

It will be useful for the ensuing discussion to keep the following graphical interpretation in mind. Since ξ is a simple linear function of δ, we can easily view (22) and (23) as functions of ξ instead of δ. This turns out to be more convenient. We will write (23) as

$$f(\xi) = g(\xi)h(\xi)V - d \qquad (25)$$

where

$$g(\xi) = \frac{\xi}{2a(a-2\xi)(\xi-a)} \qquad (26)$$

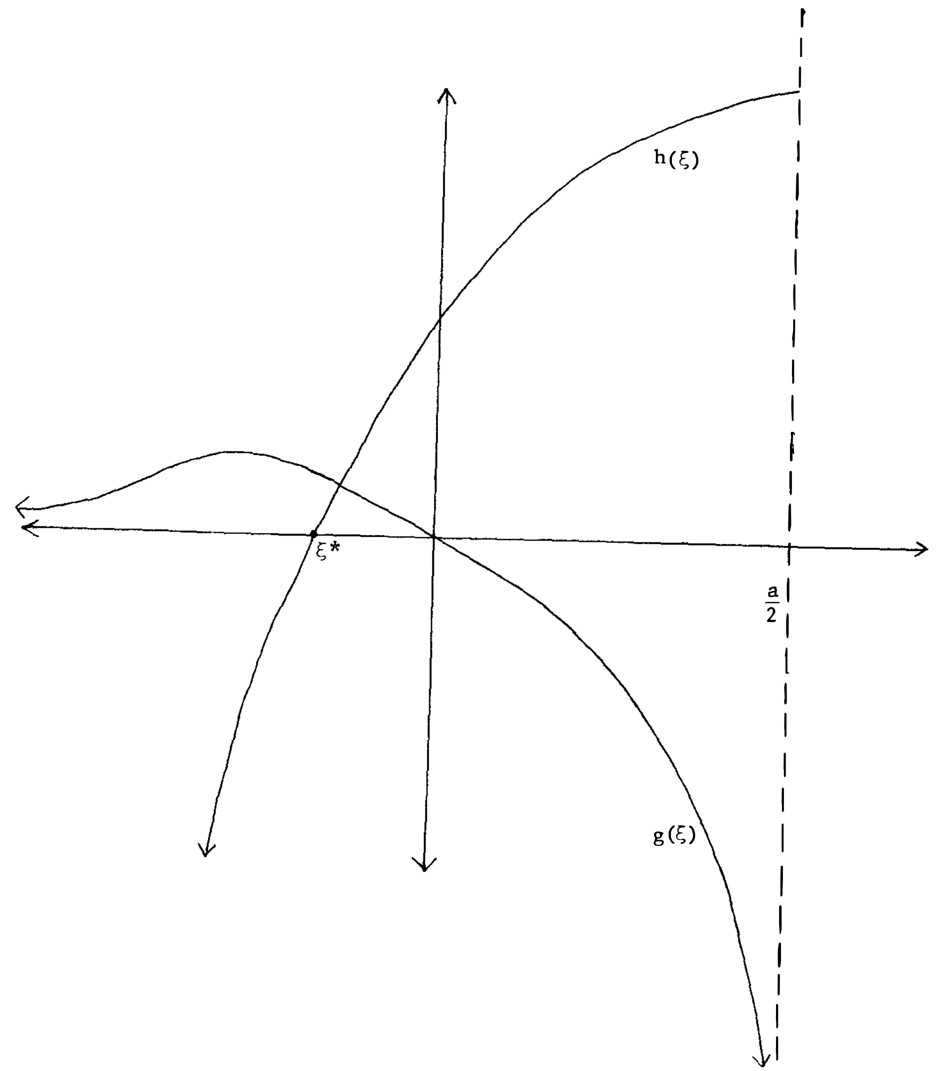
and

$$h(\xi) = 2a - c\xi(\xi - a). \qquad (27)$$

It should be clear from Figure 3 that expected profits net of the fixed cost or benefit of d are positive if and only if $\xi^* < \xi < 0$. Furthermore since g and h are both continuous, a point of maximum expected profits actually exists somewhere on the interval $(\xi^*,0)$. In particular, the point of maximum expected profits occurs at an unresponsive EFF.

Three points can now be made on the basis of this analysis. First, within the context of this model, Friedman's assertion is correct. Speculation which yields non-negative expected profits in the long run is stabilizing. If the

Figure 3



speculator is earning non-negative expected profits, we must have $|\mu| < 1$ or else expected profits would be infinitely negative. Friedman's assertion is true in an even stronger and more appealing form in this model. A speculator making optimal decisions based upon linear expectations in a linear model always has one of two effects. Either

(i) he stabilizes prices and may earn negative zero or positive expected profits.

(ii) he causes a hyperinflation and earns infinite negative profits in the limit.

That is, a speculator making optimal decisions in our model never can generate a finite increase in the variance of prices in the long run; he either reduces variance or causes a hyperinflation.

Second, expected profitability is not monotonely related to stabilizing effect. That is, a speculator earning more profits is not necessarily more stabilizing. The existence of positive profits implies that the speculator is stabilizing, but no further relation exists. To see this it is sufficient to prove that the point of maximum profits does not necessarily coincide with the point of maximum stabilizing influence.

Corollary 2: The value of ξ generating maximum expected profits is not necessarily equal to $-\frac{1}{c\beta}$ which is the value of ξ generating maximum stability.

Proof: It is sufficient to show that a case exists where $\xi^* \geq -\frac{1}{c\beta}$ because then the value of ξ generating maximum expected profits is definitely larger than $-\frac{1}{c\beta}$. The number ξ^* is the solution of $h(\xi) = 0 \wedge \xi < 0$. From Figure 3, it is sufficient to identify a case where

$$h(-\frac{1}{c\beta}) < 0$$

Now

$$h(-\frac{1}{c\beta}) = 2a - \frac{1}{\beta}(\frac{1}{c\beta} + a)$$

Since $\beta < 1$, it is clear that $c < \frac{1}{a}$ implies that $h(-\frac{1}{c\beta}) < 0$.

□

The third point is once again in a negative vein. We might expect that expectations which are more accurate[8] would generate higher expected profits. This turns out to be untrue.[9] A speculator whose expectations are more correct might actually earn smaller average profits. This seemingly nonintuitive result is explained by realizing that the speculator is perfectly competitive; he acts on the false assumption that his actions do not affect price. Given this false assumption, it is not so surprising that coupling it with another false assumption may yield better results than coupling it with a correct assumption. To prove this assertion, it is

clearly sufficient to show that cases exist where the stable rational expectations equilibrium is not the point of maximum expected profits.

Corollary 3: The stable rational expectations equilibrium is not necessarily the point of maximum profits.

Proof: The strategy of the proof will be to assume we have a point ξ^{**} which is associated with a rational expectations equilibrium and which is a point of maximum profits. We then derive the contradiction for a particular case, that $\xi^{**} \leq \xi^*$. Since ξ^{**} is a rational expectations equilibrium it satisfies (19). Since it is a point of maximum profits it satisfies (24). Putting these together, ξ^{**} must satisfy

$$(\xi - a)\xi = \frac{a}{\beta}. \tag{28}$$

Suppose that $\beta = \frac{c}{2}$. Then $h(\xi) = 0$, the equation defining ξ^*, becomes

$$(\xi - a)\xi = \frac{a}{\beta}. \tag{29}$$

Therefore $\xi^{**} = \xi^*$ which contradicts ξ^{**} generating maximum profits.

□

VII. CONCLUSION

Within the context of our model speculation which on average yields non-negative profits is stabilizing. In fact, a speculator making optimal decisions with a given method of forming expectations can never generate a finite increase in the variance of prices in the long run; he either reduces variance or causes a hyperinflation exhibiting infinite variance and negative infinite profits in the limit. A rational expectations equilibrium of each type exists. Profitability, stability and correctness are mutually unrelated in any monotone fashion. However a more responsive speculator always stabilizes prices less. If a speculator's expectations are responsive enough, he will cause a hyperinflation and destabilize prices.

The need to relax the linearity assumptions, to endogenize the choice of the EFF, and to deal more effectively with the non-negative inventories constraint has already been commented on. One other obvious direction for research concerns allowing two or more speculators using different EFF's.

FOOTNOTES

1. Friedman [2].

2. Essentially, we will assume that benefits are large enough to guarantee that some inventories will always be held. See section IV.

3. Kaldor [6].

4. Working [12].

5. The subscript t will always denote the value of the variable during period t.

6. See Goldberg [4] for a treatment of this type of difference equation.

7. As will be seen in section VI, it is the best not only in the sense that it is correct but it also maximizes profits among the class of expectations formation mechanisms of the form $E_t = \varepsilon$.

8. Recall the definition of accurate is given in section VII.

9. This is trivially true if we allow expectations which generate hyperinflations. The content of this theorem is that it is also true within the class of stabilizing expectations.

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